Résumé : We prove uniqueness of ground state solutions Q for the nonlinear equation $(-\Delta)^s Q + Q - Q^{\alpha+1} = 0$ in 1D, where $0 < s < 1$ and $0 < \alpha < \frac{4s}{1-2s}$ for $s < 1/2$ and $0 < \alpha < \infty$ for $s \geq 1/2$. Here $(-\Delta)^s$ is the fractional Laplacian. As a technical key result, we show that the associated linearized operator is nondegenerate, in the sense that its kernel is spanned by Q'. This solves an open problem posed by Kenig, Martel and Robbiano. The talk is based on joint work with E. Lenzmann.