

Problem set 2: Statistical properties of the uniform random network

Ex.1. For a random graph $G(N,p)$, where the probability that there is a link between any two pairs of nodes is $p=cst$.

- a) Calculate the expected value of the number of links $\langle m \rangle$
 b) Knowing that for one graph $G(N,p)$ of exactly m edges, the average degree of any node is

$\bar{k} = \frac{2m}{N}$, show that the expected value of the degree in the ensemble of graphs $G(N,p)$ is $\langle k \rangle = (N-1)p$. This is the expected value of a distribution of k , which one?

- c) Show that for $N \rightarrow \infty$, when $\langle k \rangle = cst$, the degree distribution of the graph is a Poisson law.

Ex.2. a) For an uncorrelated random network, show the detailed balance condition:

$$k P(k) P(k'|k) = k' P(k') P(k|k')$$

Where $P(k'|k)$: probability of a node to be connected to a node of degree k' , knowing that its degree is k :

- b) From the detailed balance condition, show that:

$$P(k'|k) = \frac{k'}{\langle k \rangle} P(k')$$

- c) Show that the average of the second neighbours of a node of degree k is:

$$\langle k_{nn} \rangle = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

Which is the assortativity type of this network?

- d) Show that the clustering coefficient of the random network is:

$$C = \frac{1}{N \langle k \rangle^3} (\langle k^2 \rangle - \langle k \rangle)^2$$

What is the scaling behaviour with the size of the system?