# Problem set 1: Network's characterization

Ex.1. For the following undirected graph:

a) Write down the adjacency matrix and the adjacency list.

b) Calculate the sparseness

c) Obtain the degrees k for all the vertices, and <k>.

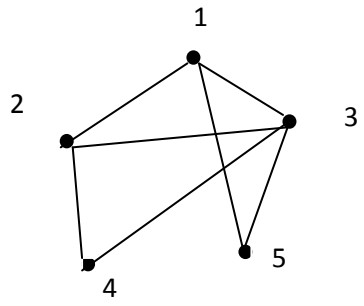
d) Calculate the diameter, $d_G$, radius $r_G$, and average shortest path length <l>,of this graph.

e) Calculate the number of shortest paths of length 2

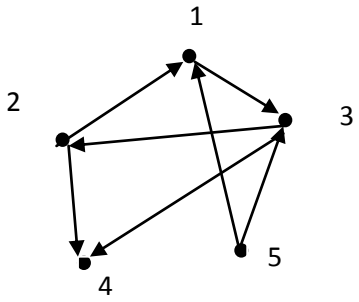
f) Calculate the closeness centrality of each node.

g) Calculate the betweenness centrality of nodes 1 and 3. Estimate how a trivial algorithm that calculates the betweenness, will scale with the size of the graph.

h) Calculate the clustering coefficient of the graph.




Ex.2. For the following directed graph:



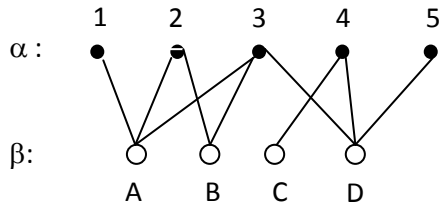

a) Write down the adjacency matrix.

b) Obtain the degrees $k_{in}$, $k_{out}$, and k for all the vertices.

Ex.3. Consider a bipartite network, with two types of vertices, and suppose there are $n_1$ vertices of type 1 and $n_2$ vertices of type 2. Show that the mean degrees, called $c_1$ and $c_2$ respectively verify:

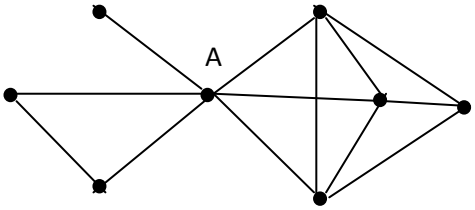
$$c_2 = \frac{n_1}{n_2} c_1$$

Ex.4. Consider the following bipartite matrix:




a) Write down the bipartite adjacency matrix.
b) Using the adjacency matrix, obtain the projections on the spaces of sets $\alpha$, and $\beta$.
c) The obtained projections are weighted networks in the corresponding spaces. Obtain the strength of each node.

Ex. 5. Consider the following undirected network:




a) Is it a planar network?
b) What is the diameter of the network?
c) Calculate the closeness centrality of vertex A
d) Calculate the unnormalized betweenness centrality of vertex A

Ex.6. A directed acyclic network is one containing no directed loops of edges

a) Give two examples from different areas of study of real-world directed networks that are acyclic or approximately so.
b) If **A** is the adjacency matrix of an acyclic network, justify that $\mathbf{A}^n = 0$, where *n* is the number of vertices in the network.