Problem set 3: Generating Functions and the Configuration Model

Ex.1. Obtain the analytical expression of the generating function of the Poisson distribution.

Ex.2. Prove the following expression: $< k^m > = \left.\frac{d^m g(z)}{d(lnz)^m}\right]_{z=1}$

Ex.3. Using the excess degree distribution, prove that for the configuration model, the clustering coefficient may be written as:

$$C = \frac{1}{N}\frac{\left[<k^2>-<k>\right]^2}{<k>^3} \quad ,$$

where N is the size of the network and <k>, $<k^2>$ are the first and second moments of the degree distribution p_k respectively.

Though this expression is formally the same that one obtains for the uniform random graph, does it lead to the same consequences concerning the dependence of C on the size of the system? Why?

Ex.4. Obtain the expression of the generating function, $g_1(z)$ for the excess degree distribution q_k of the Poisson distribution p_k.

Ex. 5. Using the "power" property of generating functions prove that the generation function

$$g^{(2)}(z) = \sum_{k=0}^{\infty} p_k^{(2)} z^k$$

of the probability, $p_k^{(2)}$, that a randomly chosen vertex has exactly k second neighbours, is: $g^{(2)}(z) = g_0(g_1(z))$

Ex.6. Consider the following distribution function: p_0, p_1, p_2,p_3 >0 and p_k=0 $\forall$ k>3.

a) Obtain the generating functions of $g_0(z)$ and $g_1(z)$ of p_k and of the excess degree distribution q_k, respectively.
b) Show that the condition for a giant component to exist is: $p_3>p_1/3$.
c) Obtain the size of the giant component S in terms of the distribution p_k.