Ex 1. Let $R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ and $S_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$.

Compute the matrix products: $R_\theta R_{\theta_2}$, $R_\theta S_{\theta_2}$, $S_{\theta_1} R_{\theta_2}$ and $S_{\theta_1} S_{\theta_2}$.

Give a geometrical interpretation.

Ex 2. Find all possible finite subgroups of $SO_2$ and of $O_2$.

Ex 3. Let $\tau = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Compute matrix $e^{\theta \tau}$, for $\theta$ some real number.

Reobtain this result computing $\lim_{n \to \infty} (I + \frac{\theta}{n} \tau)^n$. Give a geometrical interpretation.

Ex 4. Express the reflection over the line $x + y + 3 = 0$ of the usual Euclidean plane as the composition of a translation and an element of $O_2$.

Ex 5. Let us consider the transformation of the usual Euclidean plane $f(x, y) = (\frac{3}{4}x + \frac{4}{5}y - 14, \frac{1}{5}x - \frac{2}{5}y + 3)$. Check that $f$ is an isometry. Find a decomposition of $f$ as a product of translation, rotation around the origine of the plane and reflection in a line containing the origine of the plane (at most one transformation of each type).

Ex 6. Let $A = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{pmatrix}$.

Complete the entries in this matrix such that $A$ belongs to $SO_3$ and complete (in another way) to get an elements of $O_3 \setminus SO_3$. Give a geometrical interpretation.

Ex 7. Let $A = \begin{pmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \\ -1/3 & -2/3 & 2/3 \end{pmatrix}$.

Show that $A \in SO_3$. Give its geometrical elements.

Ex 8. Write the matrix in the canonical basis of $\mathbb{R}^3$ of the rotation around the axis $(O, \vec{n})$ with $\vec{n} = (1, 1, 1)$ of angle $\pi/3$.

Ex 9. Let $E$ be the following set: $E = \{ (\begin{pmatrix} z \\ w \end{pmatrix}, \theta) : \theta, (z, w) \in \mathbb{C}^2, |z|^2 + |w|^2 = 1 \}$.

For $A \in E$ compute $A \bar{A}^t$ and recognize $E$.

Give a parametrization of $SU_2$.

Ex 10. Let $\phi : \begin{cases} SU_2 & \to SO(3) \\ U & \mapsto f : (x, y, z) \mapsto (x', y', z') \end{cases}$ with $\begin{pmatrix} z' \\ x' + iy' \\ x' - iy' \end{pmatrix} = U \begin{pmatrix} z \\ x + iy \\ -z \end{pmatrix} U^{-1}$.

Show that $\phi$ is a group homomorphism. Compute $Ker \phi$. Evaluate $\phi(A)$ and $\phi(B)$ with

$A = \begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{pmatrix}$ and $B = \begin{pmatrix} \cos \theta/2 & i\sin \theta/2 \\ i\sin \theta/2 & \cos \theta/2 \end{pmatrix}$.

Conclude that $\phi$ is surjective.