Stochastic systems in mathematics and mathematical physics

University of Cergy-Pontoise, 27 January 2012

Alexandre Boritchev (École Polytechnique): *Turbulence pour l'équation de Burgers en 1D : un cas modèle pour la théorie de Kolmogorov*

Abstract: La théorie de Kolmogorov contenue dans ses 3 célèbres articles écrits en 1941 (K41) est en quelque sorte le point de départ pour tous les modèles de la turbulence. Cependant, les prédictions dans K41 ou dans les corrections à ce modèle pour les quantités statistiques à petite échelle n'ont pas pu être confirmées ou infirmées analytiquement, du fait de l'immense complexité du problème.

Ici, nous nous intéressons au modèle simplifié le plus connu pour l'équation de Navier-Stokes 3D: l'équation de Burgers stochastique. Pour ce modèle, nous estimons de façon exacte les quantités statistiques à petite échelle analogues à celles de la théorie de la turbulence. Nous confirmons notamment des résultats de l'article de Aurell, Frisch, Lutsko et Vergassola écrit en 1991 en suivant certains arguments de celui-ci.

Freddy Bouchet (ENS Lyon): *Invariant measures of the 2D Euler and 2D Navier–Stokes equations. Theoretical construction and physical applications*

Abstract: For the 2D Euler equations, we discuss the construction of the equilibrium invariant measures (with no fluxes), and the expected properties of the non-equilibrium invariant measures (inverse energy cascade and direct enstrophy cascade). We will describe the three main regimes of the 2D Navier–Stokes with stochastic forces on a torus and the expected properties of the related invariant measures. We especially discuss the relation between the invariant measures of the 2D Euler equations and Navier–Stokes equations.

One of the main related physical phenomena is the self organization of two-dimensional and geophysical turbulent flows. It is of paramount importance for atmosphere and ocean dynamics. We briefly discuss recent applications.

Charles-Edouard Brehier (ENS Cachan-Bretagne): *Strong and weak order in averaging for SPDEs, and HMM discretization scheme*

Abstract: We show an averaging principle for systems of stochastic evolution equations of parabolic type, with a small parameter ε. The fast equation is driven by an additive space-time white noise, and is assumed to be dissipative. As in the case of SDEs, we can prove the convergence of the slow component to the solution of the so-called averaged equation.

We prove that the strong order of convergence is 1-approximation of the trajectories, while the weak order is 1/2-approximation of the laws.

Knowing the speed of convergence gives important information for the derivation of efficient numerical approximation methods. We propose a scheme for the discretization in time of the system, based on the Heterogeneous Multiscale Method: we use a fast and a slow time steps, in order to deal with the smallness of ε. We explain how the averaging result can be generalized at the discrete time level, and we give explicit and efficient bounds of convergence.

Giambattista Giacomin (Paris VII): *Active rotator models*

Abstract: I will present and discuss some recent results on the long time behavior of the Fokker-Planck PDEs that arise in the study of a class of systems—known as active rotator models—of (infinitely many) noisy interacting plane rotators. Active rotator models have been introduced in biology and bio-physics and they

are a natural (non-reversible) generalization of classical rotator models in statistical mechanics. The twofold aim of the talk is to give an introduction to the rich active rotator phenomenology and to present a technique to infer results on this class of non-reversible systems by exploiting the strong control one has on a suitable underlying reversible dynamics.

Massimiliano Gubinelli (Paris IX): *Controlled paths and regularization in (S)PDEs*

Abstract: Using ideas from rough path theory we analyze some regularization phenomena in (S)PDEs. Either well-posedness or uniqueness will be enforced by adjoining to the equations some further natural conditions like a specific behaviour over small time increments. We will consider the examples of the stochastic transport equation perturbed by a Brownian noise, of the 1d Schrödinger equation with random dispersion, of the 1d deterministic periodic Korteweg–de Vries equation with rough inital data and eventually of the Kardar–Parisi–Zhang equation.

Programme

09h30-10h00 Accueil
10h00-10h05 Présentation de la journée
10h05-10h55 Freddy Bouchet
11h00-11h50 Massimiliano Gubinelli
12h15-13h45 Déjeuner
14h00-14h50 Giambattista Giacomin
14h55-15h45 Alexandre Boritchev
15h45-16h00 Pause café
16h00-16h50 Charles-Edouard Brehier